Verifying the Final Velocity of a Steel Ball Rolling on an Inclined Surface

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Abstract:

This paper is a discussion of findings of a study of motion of a steel ball rolling on an inclined surface. The goal of this paper is to prove experimentally the final velocity of the ball in order to demonstrate the conservation of energy laws.

THEORY:

We are concerned here with the laws of the conservation of energy. The conservation of energy states that any energy input into the system is exhausted in the same amount. There is no extra energy left over when the system completes its motion. In this scenario, we see that the ball beginning at rest at the top of the incline has no energy but once released, experiences motion through the force of gravity. The energy in the motion becomes the kinetic energy of the ball. We see that energy can change forms, but cannot be created or destroyed.

This experiment will accomplish three things: 1) To derive an expression for the final velocity V_f of the rolling ball, 2) to experimentally calculate V_f, and 3) to repeat for three masses to verify our results. It is our goal to verify the conservation of energy in a system and to experimentally verify the theoretical properties of V_f. One of our questions is what causes rolling of the ball as opposed to sliding down the surface. Here friction does no work, no is energy really conserved for this system?

EXPERIMENTAL METHOD:

This experiment studies the motion of a rolling steel ball down an inclined track. The track has a grooved surface which must be taken into account when rendering the equations of motion for this system. We begin with a discussion of how we derived our apparatus to allow for an accurate test of the conservation of energy. The experimental apparatus is shown in Figure 1:

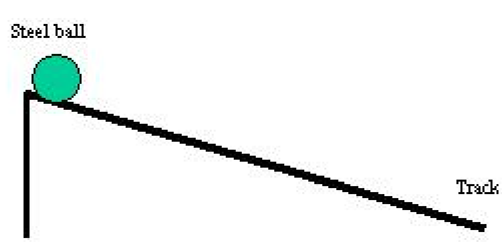

Figure 1 : Side view of ball rolling down track

The equations of this system we are testing are:

$$mgh = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2 \tag{1.1}$$

Where I is the impulse which:

$$I = \frac{2}{5}mR^2 \tag{1.2}$$

because we are dealing with a sphere. Now we must solve for v which gives us:

$$V_f = \sqrt{\frac{gh}{Q}} \tag{1.3}$$

where Q is a function of the radius R and the length l. In order to calculate the correct R for the equations, we must analyze the steel ball on the grooved track and try to calculate an R that is representative of an accurate measurement of R. In Figure 2 we see that the value of R does not lie on the bottom surface of the sphere but at a point higher on the belt. This causes us to have to calculate R by using the Pythagorean method.

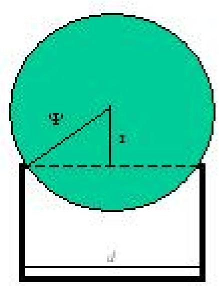

Figure 2: Finding corrected radius for the steel ball

To find $f(R, r, l)$ we break up the ball into a triangle where Ψ is the new value of R. The new quantity yields:

$$\Psi = \left[r^2 + \left(\frac{1}{2}l\right)^2\right]^{\frac{1}{2}} \tag{1.4}$$

Now rearranging equation 1.3 to incorporate the necessary variables of the system yields the theoretical value of V_f:

$$V_f = \left[0.5 + \frac{0.2\Psi^2}{\Psi^2 - \frac{l^2}{4}} \right]^{\frac{1}{2}} \qquad (1.5)$$

DATA GATHERING:

Before beginning we must note a particular problem we had with the apparatus in measuring the true belt of the sphere. The photo-gate that was to measure the quantity had to be eyed into place to capture the data. There may be more inaccuracies from this point, but we feel they are negligible when considering the goals of this experiment.

We used *DataStudio* to gather the data from our apparatus. In fifteen trials we measured three different steel spheres of different masses. We measured the time it took for the ball to travel from the top of the incline to the photo-gate at the bottom. *DataStudio* compiled the data. The numbers we used were:

$$\begin{aligned} &l = 1.0cm \\ &d_{track} = 90cm \\ &h_1 = 16.8cm \\ &h_2 = 7.7cm \\ &\textit{Where the radii of the spheres are:} \\ &\Psi_1 = 2.53cm \\ &\Psi_2 = 1.90cm \\ &\Psi_3 = 1.25cm \end{aligned} \qquad (1.6)$$

Where $h_1 - h_2 = h_{total}$. Our data for each sphere, for each different value of Ψ are:

Data for time variable in the experiment:

Time 1	Time 2	Time 3	
1.476	1.608	1.553	
1.467	1.558	1.605	
1.435	1.568	1.582	
1.506	1.553	1.564	
1.505	1.602	1.575	
1.534	1.574	1.568	
1.487	1.577	1.575	Average

We found that by contrasting our experimental results with those found in the equations that there was a 6.30% difference in values by calculating:

$$\frac{d}{t} = 1.650 m/s \tag{1.7}$$

and dividing the difference into the average of the table. The error associated with our apparatus is calculated as functions of position measurements (X), and time measurements (t):

X: +/- 0.2cm

t: +/- 0.1s

Our error equations were:

$$V = \frac{\Delta d}{\Delta t}$$

$$\delta v = \sqrt{\frac{\partial(\Delta d)}{\Delta t} + \frac{\partial(\Delta t)\Delta d}{(\Delta t)^2}} \tag{1.8}$$

$$\partial v = v\left(\frac{\delta d}{d}\right)$$

Our sources of error lie in hand measurement of the length of the grooved track, the measurement of Ψ of the spheres, and the particular task of lining up the photo-gate to obtain the maximum belt of the sphere.

<u>CONCLUSION:</u>

This paper is the columniation of a semester's work in 2219 physics. The main point was to study the conservation of energy of this system. We did find inconsistencies with our data which pointed to extra energy of the system at the end. As it is understood, a phenomenon such as this is impossible. So we conclude that the appearance of extra energy is from the frictional coefficient of the system. It was not accounted for but appeared in the real apparatus. If this experiment were done in a non-gravitational environment, we would see many interesting effects. That being said, however, the circular logic of the conservation laws would still hold—unless you get around them by using the idea of a multi-dimensional gravitational manifold which would change things rather drastically.